EFFECT OF AIR PERMEABILITY ON HEAT-INSULATING PROPERTIES OF POROUS ENVELOPES

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This paper shows that the total heat loss through a closed semipermeable envelope is equal to the sum of the purely conductive heat fluxes through any closed isothermal surface within the envelope itself. An approximate method of determining the heat loss through a closed semipermeable envelope of any form is described.

We consider a closed semipermeable envelope of porous material containing heat sources. Liquid filters through the membrane under the action of a pressure gradient, which may be due to a difference in dynamic pressures (motion of the body in a liquid or flow of liquid over the body) or to a difference in static pressures. There are no sources or sinks of liquid within the envelope. The resistance to filtration conforms to the Darcy law

$$\vec{w} = -k\vec{\nabla}p. \tag{1}$$

The velocity field in the envelope is a potential field, which follows directly from (1). We make the following additional assumptions:

a) the temperatures of the filtering flow and the body matrix in the steady state are the same at each point;

b) the thermophysical constants of the envelope material and the liquid are independent of the temperature, and those of the envelope are also independent of the spatial coordinates;

c) the temperature of the outer surface of the envelope is constant over the entire surface.

A general differential equation for combined conductive and convective heat transfer within a porous body was obtained in [1] and for the steady state can be written as:

$$\operatorname{div}\vec{q} + \sum_{k=1}^{4} c_{pk}\vec{j}_k\vec{\nabla}t = 0. \tag{2}$$

Here $\vec{j}_k$ is the mass flux density of the k-th component of the liquid. We consider a single-component incompressible liquid. Then $\vec{j}_k = \gamma_k\vec{w}_k$ and (2) takes the form

$$\operatorname{div}\vec{q} + c_p\gamma\vec{w}\vec{\nabla}t = 0. \tag{2'}$$

This equation is the condition for the absence of heat sources and sinks within the envelope.

We take any closed surface S lying entirely within the envelope itself. Integrating (2') over the volume V, bounded by the surface S, we have

$$\int_V \operatorname{div}\vec{q}dV + c_p\gamma\int_V \vec{w}\vec{\nabla}tdV = Q. \tag{3}$$

Here Q is the total intensity of the heat sources situated inside the envelope, i. e., the heat loss of the insulated volume.

It was taken into account in the integration that Eq. (2') is valid only within the insulating envelope, whereas the volume V over which the integration is taken includes the whole insulated volume as well as

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the envelope. The insulated volume contains heat sources, which compensate for the steady heat loss through the insulation.

We introduce a velocity potential φ so that $\vec{w} = \vec{\nabla}\varphi$. It is known from hydrodynamics that the velocity potential is a harmonic function [5]. We assume that in the insulated volume the motion of the liquid is also potential and on the inner boundary of the envelope $\vec{w} = \vec{\nabla}\varphi$ has no discontinuity. In this case we can assume that the velocity potential is a function φ, which is harmonic throughout the volume bounded by the outer surface of the envelope. Within the envelope $\varphi = -kp$, according to (1).

The second integral on the left side of (3) can be written, according to the first Green formula, as

$$c_p\gamma \int_V (\vec{\nabla}\varphi\,\vec{\nabla}t)\,dV = c_p\gamma \oint_S t\vec{\nabla}\varphi\,\vec{ds} - c_p\gamma \int_V t\Delta\varphi dV. \tag{4}$$

Here $\Delta\varphi = \operatorname{div} w = 0$ from the continuity equation for an incompressible liquid, and the second integral on the right side of (4) is zero. Hence,

$$c_p\gamma \int_V \vec{w}\vec{\nabla}t dV = c_p\gamma \oint_S t\vec{w}\,\vec{dS}. \tag{5}$$

In the case where the surface S is an isothermal surface (in particular, the outer surface of the envelope is isothermal), we can write

$$\oint_S t\vec{w}\vec{dS} = t \oint_S \vec{\nabla}\varphi\,\vec{dS} = 0. \tag{6}$$

The integral over a closed surface of the gradient of a function that is harmonic at all points of the volume within this surface is zero by virtue of a general property of harmonic functions [2].

From the Gauss divergence theorem

$$\int_V \operatorname{div}\vec{q}dV = \oint_S \vec{q}\vec{dS} \tag{7}$$

and Eq. (3) takes the form

$$\oint_S \vec{q}\,\vec{dS} = Q. \tag{8}$$

Thus, the steady heat loss of an insulated volume in the case where a cooling liquid filters through the heat-insulating envelope is equal to the purely conductive heat flux through the outer surface (in the general case through any isothermal surface enclosing the heat sources).

The filtration of liquid through the envelope affects the temperature distribution, according to Eq. (2'), and thus affects the heat loss. In equation (2') $\vec{q} = -\lambda\vec{\nabla}t$, and from (2') in the general case we obtain a differential equation for the temperature distribution in the envelope

$$-\lambda\Delta t + c_p\gamma\vec{w}\vec{\nabla}t = 0. \tag{9}$$

Here λ is the thermal conductivity, which includes all kinds of heat transfer through the envelope matrix and through the gaseous medium in the pores [3, 4].

We now consider a very simple specific problem. We have two plane-parallel, infinitely long walls each with a thickness δ. An air flow is directed onto the left wall, perpendicular to it, and the air filters through due to the velocity head, which has the same value p_d over the whole wall surface. The pressure drop over the thickness of the left and right walls is the same (p_d); the pressure in the enclosed volume is taken to be zero. All the temperatures are constant over the wall surfaces. The temperature notation is illustrated in Fig. 1.

The equation for steady-state filtration is the Laplace pressure equation [5] and in the given unidimensional case has the form

$$\frac{d^2p}{dx^2} = 0.$$

Hence, the filtration velocity, according to the Darcy law, will be a constant over the whole wall thickness and will be given by $u = kp_d/\delta$. For a unidimensional flow Eq. (9) can be put in the form:

$$\frac{d^2t}{dx^2} - \mu\frac{dt}{dx} = 0, \tag{10}$$

where $\mu = c_p\gamma u/\lambda$.

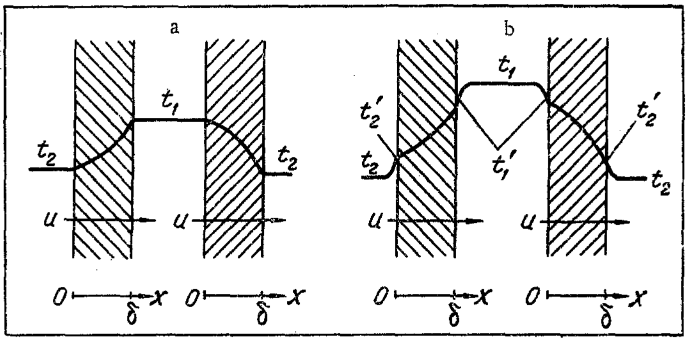

Fig. 1. Boundary conditions for Eq. (10). Filtration velocity: a) high; b) low.

The unidimensional Eq. (10), which gives an exponential temperature distribution in the wall, was obtained in [7] from an analysis of the heat transfer between the liquid in the pores and the matrix and from the heat balance of an element of wall thickness [8]. It should be noted that investigations [7, 8] related to problems of aviation and rocket technique (porous cooling of a metal wall operating at maximum temperature). Hence, the authors of [7, 8] found the temperature distribution in order to determine the thermal stresses in the wall, and did not analyze the heat fluxes.

Solutions of Eq. (10) are given in [7, 8] for boundary conditions of the first and third kind (Fig. 1a and b), but these solutions are not applicable in the given case. The fact is that the authors of [7, 8] used Weinbaum and Wheeler's results [6], according to which the temperatures of the metal and liquid on the surface on the coolant injection side in the case of porous cooling of a metal wall are significantly different, but these temperatures are practically the same throughout the thickness of the wall. The distance at which there is a finite difference between the temperatures of the solid matrix and liquid is very small (of the order of the pore diameter [7]). Here we are not dealing with metal walls, but with heat-insulating walls, in which the thermal conductivity of the solid matrix is low, and there is often poor contact between the elements of this matrix. Hence, in the conditions of the given problem we can assume that the temperatures of the filtering liquid and the solid body are equal at all points without exception, including those on the entry side of the wall.

In the analysis of the boundary conditions for Eq. (10) we consider two limiting cases. We assume firstly, that the filtration velocity is so great that the whole thermal boundary layer formed on the entry surface of the envelope is "sucked" into the envelope. A thermal boundary layer cannot be formed on the exit surface in the same way. In this case the temperatures of the boundary surfaces will be equal to the corresponding temperatures of the surroundings (boundary conditions of the first kind, Fig. 1a). These are also the boundary conditions for the case of filtration due to action of a static pressure difference [9]. The second case occurs when the permeability of the envelope is relatively low and has practically no effect on the formation of the boundary layer on the wall. In this case we can assume that the heat transfer coefficients are the same as for the case of an impermeable envelope. The given temperatures here are the temperatures of the surroundings, which are not equal to the temperatures on the surface (boundary conditions of third kind, Fig. 1b). These are also the boundary conditions for all the intermediate cases if the heat transfer coefficients can be assumed known.

The solutions of Eq. (10) for boundary conditions of the first kind (Fig. 1a) have the form

$$t_L = t_2 + (t_1 - t_2)\,\frac{\exp \mu x - 1}{\exp \mu\delta - 1}\,, \tag{11'}$$

$$t_R = t_1 - (t_1 - t_2)\,\frac{\exp \mu x - 1}{\exp \mu\delta - 1}\,. \tag{11''}$$

The conductive heat flux, as distinct from the case without filtration, is not constant throughout the thickness of the wall

$$|\vec{q}| = \lambda \left| \frac{dt}{dx} \right| = c_p \gamma u\,(t_1 - t_2)\,\frac{\exp \mu x}{\exp \mu\delta - 1}\,. \tag{12}$$

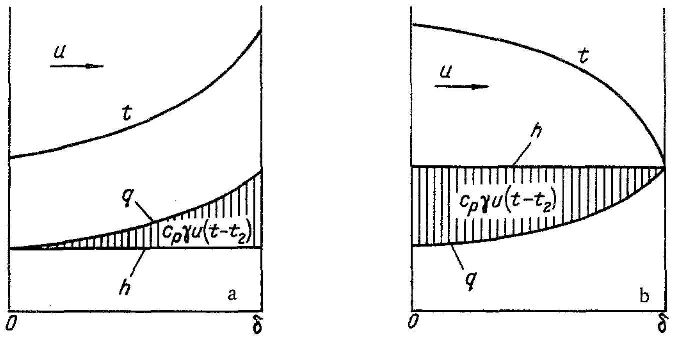

Fig. 2. Temperature and components of enthalpy flux over thickness of left (a) and right (b) walls.

Here $|\vec{q}|$ denotes the absolute heat flux, since the sign of $\vec{q}$ on the x axis will be different for the left and right walls.

Equation (2) was obtained in [1] on the assumption that the local derivative with respect to time of the volume enthalpy concentration is equal to the divergence from the enthalpy flux. Hence, in the steady-state unidimensional case the enthalpy flux, i. e., in this case the combined convective and conductive heat transfer, will be constant throughout the thickness of the wall:

$$h = q + c_p\gamma u (t - t_2) = \text{const}. \quad (13)$$

We assume the enthalpy to be zero at $t = t_2$. Substituting in (13) the values obtained above for t and q with due regard to sign, we obtain

$$h_L = -\frac{t_1 - t_2}{\exp \mu\delta - 1} c_p\gamma u, \quad (13')$$

$$h_R = \frac{c_p\gamma u \exp \mu\delta}{\exp \mu\delta - 1} (t_1 - t_2). \quad (13'')$$

The distribution of the temperature and the components of the enthalpy flux over the thickness of the left and right walls is shown in Fig. 2.

The total heat flux through an envelope composed of two walls is

$$Q = F|h_L| + F|h_R| = F(|h_L| + |h_R|).$$

We determine the specific heat loss from (13):

$$|h_L| + |h_R| = c_p\gamma u (t_1 - t_2) \frac{\exp \mu\delta + 1}{\exp \mu\delta - 1}. \quad (14)$$

It is easy to see from (12) that the sum of the conductive heat fluxes $|q_L| + |q_R|$ through the isothermal outer or inner surface has the same value.

The obtained expression (14) gives the specific heat loss through the two walls in the presence of filtration. At the limit $u \to 0$, i. e., in the absence of filtration, Eq. (14), after application of the L'Hopital rule, becomes the usual expression for the steady heat flux through an impermeable plane wall.

For low filtration velocities we must regard the temperatures of the medium t_1 and t_2 as prescribed; these temperatures differ from the temperatures on the wall surface (Fig. 1b). On the entry surface of the left wall we have $\alpha_2(t_2' - t_2) = \lambda(dt/dx)_{x=0}$. For the right wall, accordingly, $\alpha_1(t_1 - t_1') = -\lambda(dt/dx)_{x=0}$. The second boundary condition can be obtained from (13) and for the two walls is written as:

$$\alpha_1(t_1 - t_1') - \alpha_2(t_2' - t_2) = c_p\gamma u (t_1' - t_2').$$

Determining the constants of integration from these conditions, we obtain the following solutions of Eq. (10) for the temperature distribution:

$$t_L = t_2 + (t_1 - t_2) \frac{K_1 + \exp \mu x}{K_1 + K_2 \exp \mu\delta}, \quad (15')$$

$$t_R = t_1 - (t_1 - t_2) \frac{K_1 + \exp \mu x}{K_1 + K_2 \exp \mu\delta}. \quad (15'')$$

Here

$$K_1 = \frac{c_p \gamma u}{\alpha_1} - 1 \text{ and } K_2 = \frac{c_p \gamma u}{\alpha_2} + 1.$$

For the conductive heat flux we obtain

$$|q| = c_p \gamma u (t_1 - t_2) \frac{\exp \mu x}{K_1 + K_2 \exp \mu \delta}. \tag{16}$$

The total heat flux in view of (8) is given by the following expression:

$$Q = |q_L|_{x=0} + |q_R|_{x=\delta} = c_p \gamma u (t_1 - t_2) \frac{1 + \exp \delta \mu}{K_1 + K_2 \exp \mu \delta}. \tag{17}$$

We now consider a stationary closed porous envelope bounded by plane-parallel walls (n faces altogether) in a flow of liquid. We assume that the external pressure on each face is constant and can be assumed known. On the inner surfaces of the walls the pressure is the same for all walls and is equal to the pressure p_0 in the enclosed volume.

The exact solution of the Laplace equation $\Delta p = 0$ even for envelopes of the simplest configuration is obtained in the form of infinite series and from the boundary conditions for p we cannot obtain a suitable expression for the velocity $\vec{w} = -k\vec{\nabla} p$ for further manipulation. If we assume, however, that a free return flow of mass from one wall to any other is possible within the envelope, the thickness of the envelope is small in comparison with its other dimensions, then in the case of relatively low permeability of the envelope we can neglect the return flow of mass from one face to the other within the envelope itself (at the corners). The filtration velocity in each face can then be assumed constant and directed normal to the surface. The pressure within the envelope and the magnitude and direction of the velocity (inward or outward) in each face are given by the following system of equations

$$\begin{aligned} w_1 &= \frac{k}{\delta_1} (p_1 - p_0), \\ w_2 &= \frac{k}{\delta_2} (p_2 - p_0), \\ &\cdots\cdots \\ w_n &= \frac{k}{\delta_n} (p_n - p_0), \\ &\sum_{i=1}^{n} \vec{F}_i \, \vec{w}_i = 0. \end{aligned} \tag{18}$$

It was assumed in the last case that the direction of the vector $\vec{F}_i$ coincides with the direction of the external normal to the surface.

Each of the walls will have the exponential temperature distribution (11) or (15), depending on the filtration velocity and the associated method of assigning the boundary conditions. The total heat flux through the envelope, according to (8), is given by:

$$Q = \sum_{i=1}^{n} \vec{F}_i \, \vec{q}_i = \sum_{i=1}^{n} |\vec{F}_i| \, |\vec{q}_i|, \tag{19}$$

where $|q_i|$ for each wall is determined from (12) or (16) for any isothermal surface. For (16) α_i on the inner and outer surfaces of the wall on each face will be known. The x axis in each face will have the same direction as the filtration velocity and the origin will be on the entry side of the wall.

This method of approximate solution can also be used for an envelope of any configuration if the component of the filtration velocity parallel to the surface can be neglected. This simplification is justified on the basis of the assumption made above for an envelope bounded by planes.

For relatively thin hollow closed envelopes of any form through which a cooling liquid filters we can recommend the following method of determining the heat flux:

a) we replace the envelope of the given form by a hollow polyhedron;

b) from the prescribed $\alpha(S)$, $p(S)$, and $t(S)$ for the given envelope we determine p, α, and t for each face;

c) we find from (18) the magnitude and direction of the filtration velocity in each face;

d) from (12) or (16) we find for each face $|q_i|$ at x corresponding to the outer (or other isothermal) surface of this face;

e) we determine from (19) the total heat loss of the insulated volume.

This calculation can theoretically be made with any degree of accuracy if the division is sufficiently fine.

NOTATION

α is the heat transfer coefficient;
δ is the wall thickness;
F is the surface area;
h is the enthalpy flux density;
j is the mass flux density;
γ is the specific gravity of gas (air);
c_p is the specific heat of gas (air) at constant pressure;
λ is the effective thermal conductivity of porous material;
p is the static pressure;
q is the conductive heat flux density;
k is the permeability of porous material;
φ is the filtration velocity potential;
w is the filtration velocity;
u is the projection of filtration velocity on x axis;
x is the coordinate of depth in wall;
n is the number of faces of hollow polyhedron.

Subscripts

1 and 2 denote inner and outer surfaces;
L and R denote left (entry) and right (exit) walls;
i is the number of the face of the polyhedron.

LITERATURE CITED

1. A. V. Lykov, in: Heat Transfer Problems [in Russian], Atomizdat (1967).
2. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Nauka (1966).
3. A. F. Chudnovskii, Thermophysical Characteristics of Disperse Materials [in Russian], Fizmatgiz (1962).
4. G. N. Dul'nev and B. L. Muratova, Inzh.-Fiz. Zh., 14, No. 1 (1968).
5. V. I. Aravin and S. N. Numerov, Theory of Motion of Liquids and Gases in a Nondeformable Porous Medium [in Russian], GITTL (1953).
6. S. Weinbaum and H. L. Wheeler, J. Appl. Phys., 20 (1949).
7. E. Mayer and J. G. Bartas, Jet Propulsion, 24, No. 6 (1954).
8. P. J. Schneider, Jet Propulsion, 27, No. 8 (1957).
9. S. C. Hyman, Jet Propulsion, 26, No. 9 (1956).